Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Friday 7 June 2024

Morning (Time: 2 hours)

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule
In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 $f(x) = 6x^3 - 13x^2 + ax - 10$ where a is a constant

Given that $(3x - 2)$ is a factor of $f(x)$

(a) show that $a = 21$

(2)

(b) Hence show algebraically that the curve $y = f(x)$ has only one intersection with the x-axis.

(4)

(Total for Question 1 is 6 marks)

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2 The quadratic equation $3x^2 - 5x + 1 = 0$ has roots α and β

Without solving the equation,

form a quadratic equation with integer coefficients, that has roots $\frac{\alpha}{2\beta}$ and $\frac{\beta}{2\alpha}$

(8)

Question 2 continued

(Total for Question 2 is 8 marks)

3

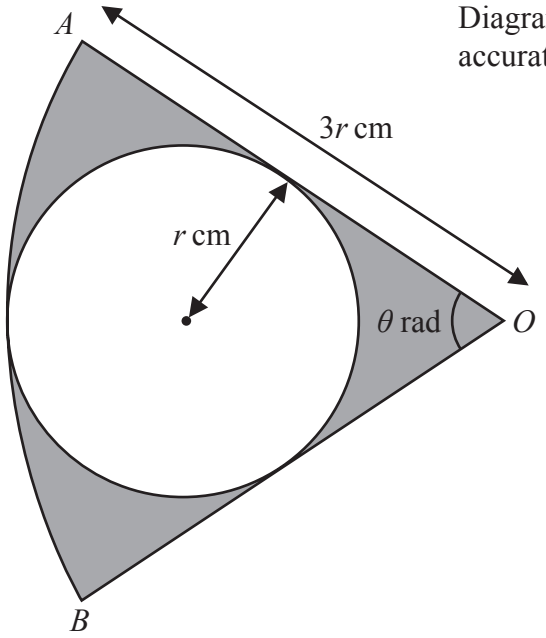

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the sector AOB of a circle with centre O and radius $3r$ cm

A circle with radius r cm touches OA and OB and the arc AB

Angle AOB is θ radians, where $0 < \theta < \frac{\pi}{2}$

(a) Find the exact value of θ

(2)

The area of the region shown shaded in Figure 1 is 8π cm^2

(b) Find the value of r

(4)

Question 3 continued

(Total for Question 3 is 6 marks)

4

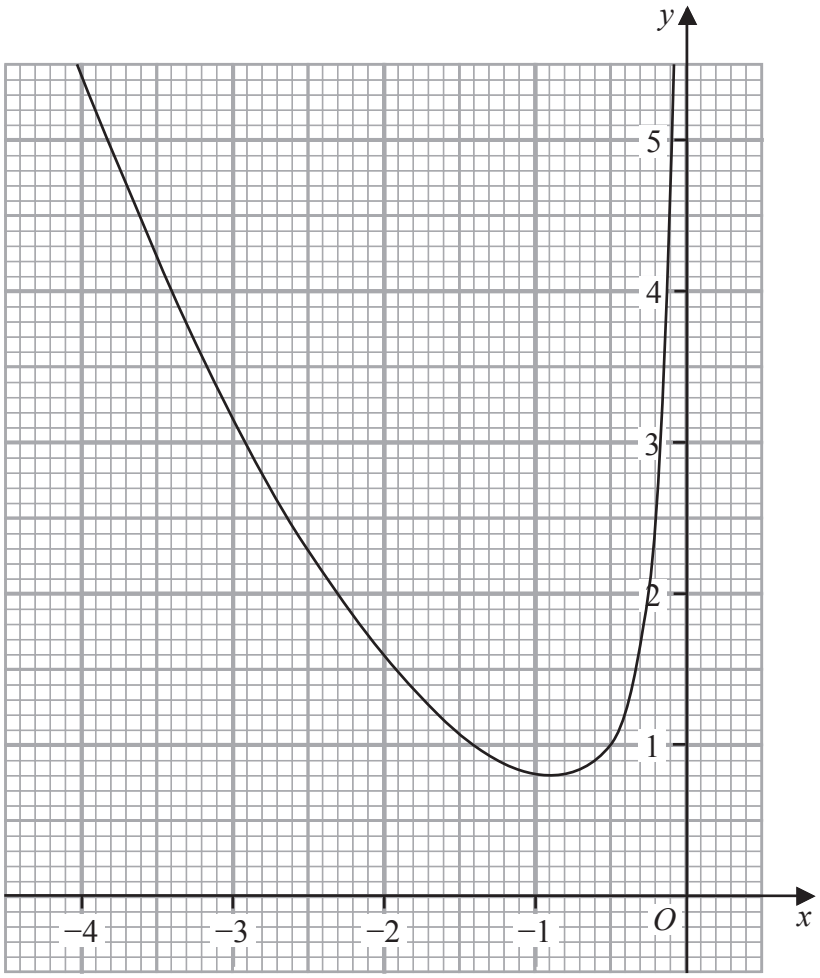

Figure 2

Figure 2 shows part of the curve with equation $y = \frac{x^2}{3} - \frac{1}{2x}$ for $-4 < x < 0$

By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $4x^3 + 3x^2 - 36x - 6 = 0$ in the interval $-4 < x < 0$

(4)

Question 4 continued

(Total for Question 4 is 4 marks)

5

$$y = e^{2x}\left(x^2 - 5x\right)$$

Show that $2e^{2x} = \dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y$

(7)

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Question 5 continued

(Total for Question 5 is 7 marks)

6

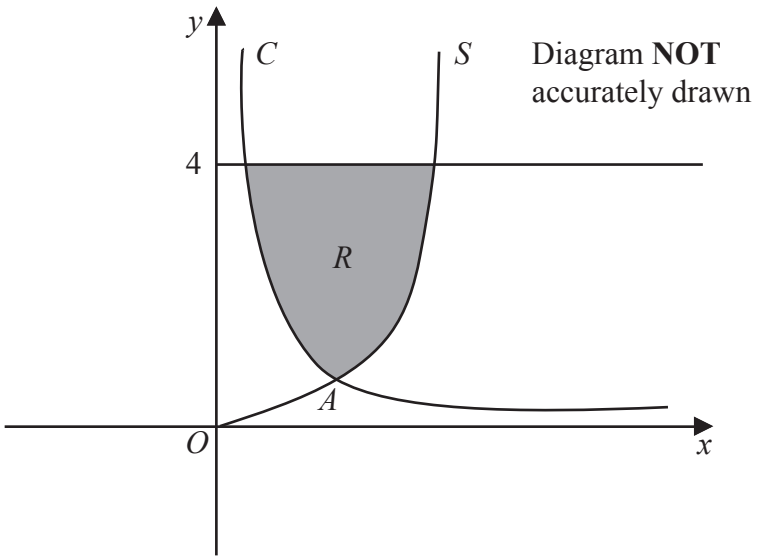

Figure 3

Figure 3 shows part of the curve C with equation $y = \frac{1}{4x}$, $x > 0$ and part of the curve S with equation $y = 2x^2, x \geqslant 0$

The curve C and the curve S intersect at the point A

(a) Find the coordinates of point A

(3)

The finite region R, shown shaded in Figure 3, bounded by the curve C, the curve S and the straight line $y = 4$ is rotated through 360° about the y-axis.

(b) Find, using algebraic integration, the exact volume of the solid formed.

(7)

Question 6 continued

(Total for Question 6 is 10 marks)

7 (a) Expand $\left(1 + 2x^2\right)^{-\frac{3}{4}}$ in ascending powers of x up to and including the term in x^6

Express each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{(2 + kx)}{\left(1 + 2x^2\right)^{\frac{3}{4}}} \quad \text{where } k \neq 0$$

(b) Obtain a series expansion for $f(x)$ in ascending powers of x up to and including the term in x^5

Give each coefficient in terms of k where appropriate.

(2)

The coefficient of the term in x^5 is fourteen times the coefficient of the term in x^2

(c) Find the value of k

(2)

Question 7 continued

Question 7 continued

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Question 7 continued

(Total for Question 7 is 7 marks)

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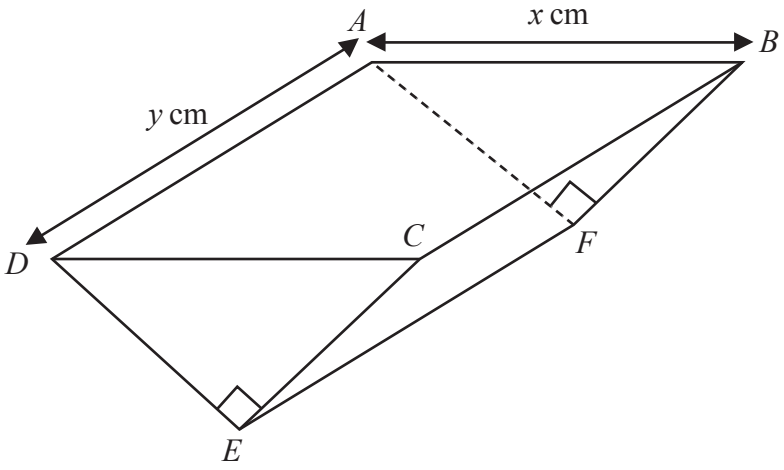

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Figure 4

Figure 4 shows a solid right triangular prism $ABCDEF$

The cross section of the prism is an isosceles triangle.

- $\angle DEC = \angle AFB = 90^{\circ}$
- $AB = DC = x$ cm
- $AD = BC = FE = y$ cm
- $AF = BF = DE = CE$

The triangular faces of the prism are vertical and the edges AD, BC and FE are horizontal.
The volume of the prism is 3.6 cm^3
The total external surface area of the prism is S cm^2

(a) Show that S satisfies the equation

$$S = \frac{x^2}{2} + \frac{72\left(\sqrt{2}+1\right)}{5x}$$

(4)

Given that x can vary,

(b) use calculus, to find to 3 significant figures, the value of x for which S is a minimum.
Justify that this value of x gives a minimum value of S

(4)

(c) Hence find, to 2 significant figures, the minimum value of S

(2)

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Question 8 continued

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

9

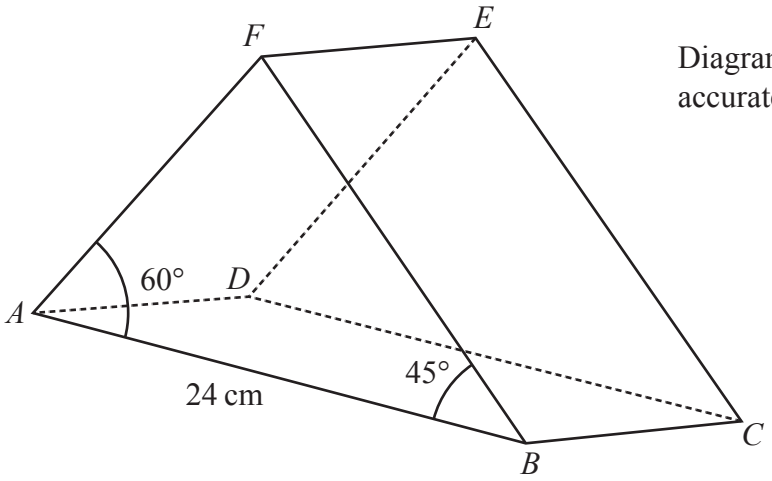

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Figure 5

Figure 5 shows a right triangular prism $ABCDEF$ where $ABCD$ is a rectangle.

$AF = DE$ $BF = CE$ $AD = FE = BC$ $AB = DC = 24$ cm

$\angle ABF = \angle DCE = 45°$ $\angle BAF = \angle CDE = 60°$

Using a formula from page 2,

(a) show that $\sin AFB = \dfrac{\sqrt{2} + \sqrt{6}}{4}$

(3)

Without using a calculator,

(b) show that $BF = 12\left(3\sqrt{2} - \sqrt{6}\right)$ cm

(5)

The angle between the plane AEB and the plane $ABCD$ is $65°$

(c) Find, in cm to 2 significant figures, the length of EF

(3)

(d) Find, in degrees to one decimal place, the size of the angle between the line CF and the plane $ABCD$

(4)

Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 15 marks)

10 The points A, B, C and D are the vertices of a quadrilateral such that

$$\overrightarrow{AB} = 3\mathbf{a} + 4\mathbf{b} \quad \overrightarrow{AC} = 7\mathbf{a} + 9\mathbf{b} \quad \overrightarrow{AD} = 4\mathbf{a} + 5\mathbf{b}$$

(a) Show that $ABCD$ is a parallelogram.

(3)

BC is extended to the point E such that BCE is a straight line.

Point F lies on CD such that $CF : FD = 1 : 2$

Given that A, F and E are collinear,

(b) find the vector $\overrightarrow{AE}$ in the form $X\mathbf{a} + Y\mathbf{b}$ where X and Y are rational numbers to be found.

(8)

Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 11 marks)

11 Using formulae from page 2, show that

(a) (i) $\cos 2A = 2\cos^2 A - 1$

(3)

(ii) $\sin 2A = 2\sin A \cos A$

(1)

(b) Show that $\cos^3 A = \dfrac{\cos 3A + 3\cos A}{4}$

(4)

Hence, or otherwise,

(c) solve, giving exact values in terms of π

$$8\cos^3\left(\frac{\theta}{2}\right) - 6\cos\left(\frac{\theta}{2}\right) - 1 = 0 \qquad \text{for } 0 \leqslant \theta \leqslant 2\pi$$

(4)

(d) use algebraic integration to find the exact value of

$$\int_0^{\frac{\pi}{6}} \left(4\cos^3\theta - \sin 2\theta\right) \mathrm{d}\theta$$

(4)

Question 11 continued

Question 11 continued

(Total for Question 11 is 16 marks)

TOTAL FOR PAPER IS 100 MARKS

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