Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
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Centre Number | Candidate Number

# Pearson Edexcel International GCSE

## Friday 7 June 2024

Morning (Time: 2 hours)

**Paper reference** **4PM1/02R**

# Further Pure Mathematics
## PAPER 2R

**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

### Series

**Arithmetic series**

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

**Geometric series**

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

**Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

### Calculus

**Quotient rule (differentiation)**

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

### Trigonometry

**Cosine rule**
In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

### Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** The quadratic equation

$$kx^2 - (2k + 6)x + 16 = 0$$

has equal roots.

Find the two possible values of $k$

**(4)**

**(Total for Question 1 is 4 marks)**

**2** (a) Expand $\frac{2}{\sqrt{1+3x}}$ in ascending powers of $x$ up to and including the term in $x^3$

Express each coefficient as a fraction in its simplest terms where appropriate.

**(4)**

(b) State the range of values of $x$ for which the expansion is valid.

**(1)**

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**Question 2 continued**

**(Total for Question 2 is 5 marks)**

**3** $O$, $A$ and $B$ are fixed points such that

$$\left|\overrightarrow{OA}\right| = 3\sqrt{5} \qquad \overrightarrow{AB} = \mathbf{i} + 3a\mathbf{j} \qquad \overrightarrow{OB} = 7\mathbf{i} + 2a\mathbf{j}$$

Given that $a > 0$

(a) find the value of $a$

**(4)**

(b) Hence find a unit vector parallel to $\overrightarrow{OA}$

**(2)**

**Question 3 continued**

**(Total for Question 3 is 6 marks)**

**4** $f(x) = px^3 + qx^2 - 37x - 12q$ where $p$ and $q$ are constants.

When $f'(x)$ is divided by $(x + 2)$ the remainder is $-33$

Given that $(x + 5)$ is a factor of $f(x)$

(a) (i) show that $p = 2$

(ii) find the value of $q$

**(6)**

(b) Hence, use algebra to factorise $f(x)$ completely.

**(3)**

(c) Hence solve the equation $f(x) = 0$

**(2)**

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**Question 4 continued**

**(Total for Question 4 is 11 marks)**

**5** The force $F$ newtons between two magnetic poles is given by the formula

$$F = \frac{3}{20r^2}$$

where $r$ is the distance, in centimetres, between the poles.

The distance between the two poles is increasing at a constant rate of 0.7 cm/s

Find the rate of change of $F$, in newtons/s to 3 significant figures, when the distance between the poles is 2.8 cm

**(6)**

**Question 5 continued**

**(Total for Question 5 is 6 marks)**

**6**

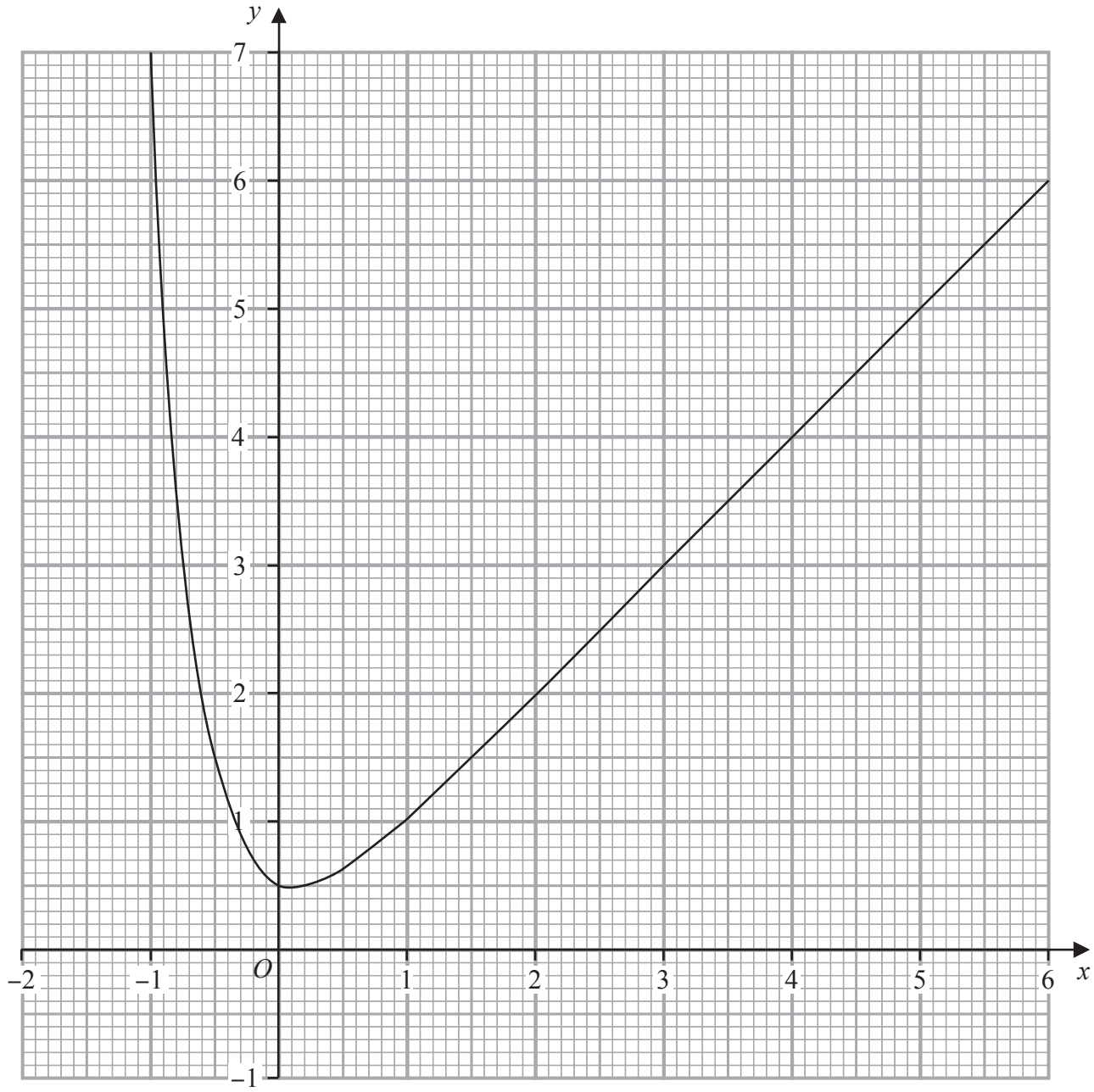


**Figure 1**

Figure 1 shows part of the graph of the curve with equation $y = x + 2^{-(4x+1)}$

By drawing a suitable straight line on the graph, obtain an estimate, to one decimal place, of the roots of the equation $\log_2(8 - 3x) + 4x = 0$ in the interval $-2 \leqslant x \leqslant 6$

**(7)**

**Question 6 continued**

**(Total for Question 6 is 7 marks)**

7

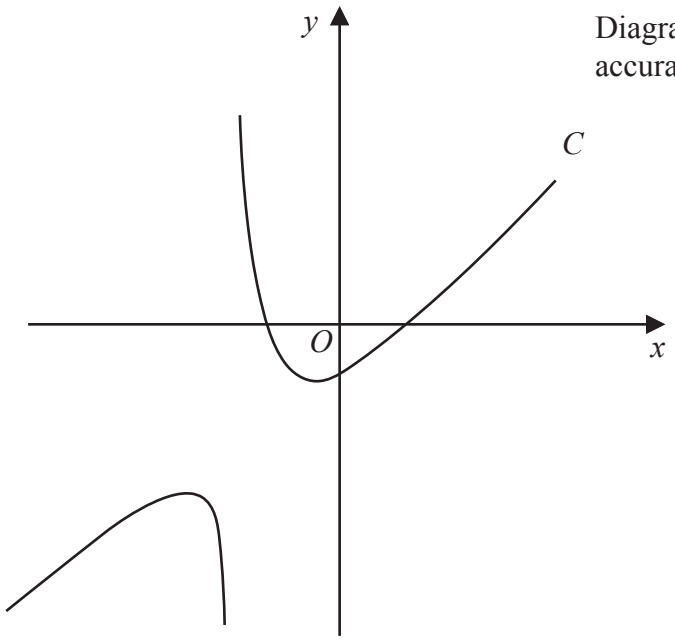


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**Figure 2**

Figure 2 shows a sketch of part of the curve $C$ with equation

$$y = \frac{x^2 - 1}{4x + 5} \quad \text{where} \quad x \neq -\frac{5}{4}$$

(a) Write down the equation of the asymptote to $C$ that is parallel to the $y$-axis.

**(1)**

The line $l$ is the normal to $C$ at the point where $x = -1$

(b) Find an equation of $l$

**(7)**

The line $l$ meets $C$ again at the point $D$

(c) Find the coordinates of $D$

**(6)**

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**Question 7 continued**

**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 14 marks)**

**8** The sum of the first 2 terms of a geometric series $G$ is 360
The sum of the 2nd and 3rd terms of $G$ is 288

The $n$th term of $G$ is $U_n$

(a) Show that $U_n = A\left(\frac{4}{5}\right)^{n-1}$ where $A$ is an integer to be found.

**(7)**

(b) Explain why $G$ is convergent.

**(1)**

(c) Hence find the sum to infinity of $G$

**(2)**

(d) Find the least number of terms for which the sum is greater than 978

**(4)**

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**Question 8 continued**

**Question 8 continued**

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**Question 8 continued**

**(Total for Question 8 is 14 marks)**

**9**

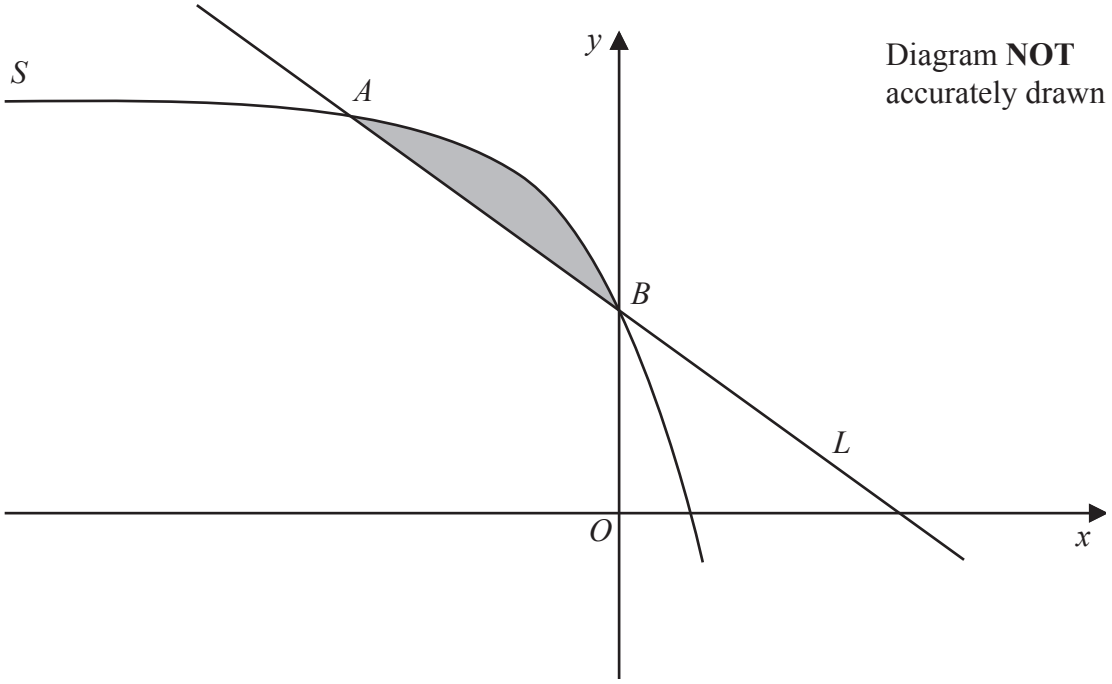


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**Figure 3**

Figure 3 shows a sketch of part of the curve $S$ with equation $y = -2e^{3x} + 4$ and the line $L$

The curve $S$ has intersections with the line $L$ at the points $A$ and $B$ with $x$ coordinates $x = -1$ and $x = 0$ respectively.

The finite region bounded by $S$ and $L$ is shown shaded in Figure 3

Use calculus to find the exact area of this region.

Give your answer in the form $\dfrac{a + be^{-c}}{c}$ where $a$, $b$ and $c$ are integers to be found.

**(8)**

**Question 9 continued**

**Question 9 continued**

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**Question 9 continued**

**(Total for Question 9 is 8 marks)**

**10**

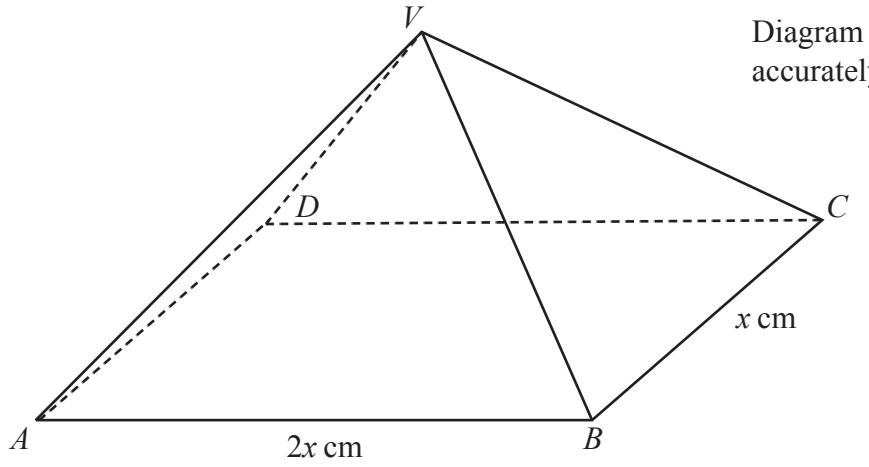


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**Figure 4**

Figure 4 shows a right pyramid $ABCDV$

The base of the pyramid is a rectangle where,

$$AB = DC = 2x \text{ cm} \quad AD = BC = x \text{ cm}$$

The edges $VA, VB, VC$ and $VD$ are all of equal length.

The angle between $VA$ and $ABCD$ is 45°

(a) Show that $VA = \frac{\sqrt{10}}{2} x$ cm

**(3)**

(b) Find in cm, the exact height of the pyramid in terms of $x$

**(2)**

Find, in degrees to one decimal place,

(c) the size of angle $VBA$

**(2)**

(d) the size of the obtuse angle between the plane $AVC$ and the plane $BVD$

**(4)**

Given that the volume of the pyramid is $9\sqrt{5}$ cm$^3$

(e) find the value of $x$

**(2)**

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**Question 10 continued**

**Question 10 continued**

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**Question 10 continued**

**(Total for Question 10 is 13 marks)**

**11** (a) Using a formula on page 2 show that $\cos 2A = 2\cos^2 A - 1$

**(2)**

(b) Hence show that $\left(2\cos^2 A - 1\right)^2 = \dfrac{\cos 4A + 1}{2}$

**(3)**

The curve with equation $y = \dfrac{\sin 2x}{2} + \dfrac{\left(2\cos^2 x - 1\right)^2}{2} + \dfrac{1}{8}$ has a stationary point $P$ in the range $0 \leqslant x \leqslant \dfrac{\pi}{6}$

(c) Find the exact coordinates of $P$

**(7)**

**Question 11 continued**

**Question 11 continued**

**(Total for Question 11 is 12 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

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